Discovering the temperature of the sun using a standard 12 GHz satellite receiver

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October 10, 2005

Abstract

This paper details the measuring of the temperature of the sun with a common commercial 12 GHz satellite receiver.

1 Introduction

The sun is a blackbody radiator. Finding the brightness temperature of the sun at any frequency therefore gives us a value which describes the physical temperature of the sun. It is therefor possible to gauge the physical temperature of the sun by measuring the brightness temperature of the sun with a common 12 GHz receiver.

2 Theory

We use the following reference temperatures

$$T_{Cmb} = 2.728 \pm 0.004K \quad (1)$$

$$T_{Ground} = 298 \pm 5K \quad (2)$$

$$T_{Human} = 310 \pm 0.5K \quad (3)$$

All observable temperatures are related to the sum of the temperature sources, multiplied by some constant of proportionality. The relationship between the observable temperatures and the contributing temperatures is given by the following equations.

$$O_{Sky} = K_{prop}(T_{CMB} + T_{EL}) \quad (4)$$

$$O_{Sun} = K_{prop}(T_{SUN} + T_{CMB} + T_{EL}) \quad (5)$$

$$O_{Ground} = K_{prop}(T_{GROUND} + T_{EL}) \quad (6)$$

$$O_{Human} = (K_{prop}(T_{HUMAN} + T_{EL}) \quad (7)$$

where T_{EL} is the inherent noise is the receiver equipment, common to all equations.

Equating equations 6 and 7 via the common constant of proportionality yields :

$$\frac{T_{EL} + T_{HUMAN}}{20.5} = \frac{T_{EL} + T_{GND}}{22} \tag{8}$$

enabling us to solve for T_{EL} using 2 & 3.

Solving for equations 4 & 7 simultaneously we have :

$$\frac{T_{EL} + T_{HUMAN}}{20.5} = \frac{T_{EL} + T_{SKY}}{7.5} \tag{9}$$

enabling us to solve for T_{EL} using 1 & 3.

Equating equations 4 and 5 via the common constant of proportionality yields :

$$\frac{T_{EL} + T_{CMB} + T_{SUN}}{18} = \frac{T_{CMB} + T_{EL}}{7.5} \tag{10}$$

enabling us to solve for T_{SUN} using T_{EL}, 1 & 3.

The sun is small in proportion to the area of sky the receiver covers. The ratio of the angular area of the sun (as viewed from earth) over the angular area of the main beam beam gives a factor which accounts for the fact that the sun does not cover the full beam of the radio receiver.

The following equation can used to discover the angular diameter of the radio receiver's main beam.

$$\theta = \frac{\lambda}{diameter} \tag{11}$$

where λ and diameter are dictated by the receiving frequency and diameter of the receiver respectively.

The angular diameter of the sun can be discovered by making a pin prick in a piece of paper, and measuring the diameter of the circle created by light shining through the hole, and the distance from the hole to the surface the light is projected on.

$$sin(\theta) = \frac{opposite}{adjacent} \tag{12}$$

$$sin(\theta) = \frac{d}{D} \tag{13}$$

$$\therefore \theta \approx \frac{d}{D} \tag{14}$$

where d is the angular diameter of the sun from earth, and D is the distance from earth to the sun.

The receiver we are dealing with is imperfect, and only has an efficiency of 70%, so our measured tempreture must be adjusted to include this consideration.

Equation 11 was a rough approximation, and is better represented by

$$\theta = \frac{1.2\lambda}{diameter} \quad (15)$$

Uncertainties are calculated using

$$\sigma_T^2 = \sigma_x^2(\frac{dT}{dx})^2 + \sigma_y^2(\frac{dT}{dy})^2 + \sigma_z^2(\frac{dT}{dz})^2 \quad (16)$$

3 Procedure

A standard 12 GHz [1] satellite receiver was pointed directly at the sun. The same receiver was then directed at the sky, the ground and a human being which functioned as known calibrators. Once the arbitrary scale was calibrated we could associate a meaningful conversion factor with the number produced by the receiver.

We used equations 8 and 9 to independently discover T_{EL}. We used equation 10 to calculate the temperature of the sun.

Values were acquired for θ_{rsun} and θ_{rbeam} by using equations 14 and 11 respectively. The ratio of θ_{rsun} to θ_{rbeam} was calculated, and used to adjust the temperature of the sun accordingly.

We corrected the measured value of the temperature of the sun by taking into account the efficiency of the receiver and also using the better approximation of angular diameter of the receiver described by equation 15.

All uncertainties were calculated is the fashion described by equation 16.

4 Results

The observed temperatures are :

$$O_{Sky} = 7.5 \quad (17)$$
$$O_{Sun} = 18 \quad (18)$$
$$O_{Ground} = 22 \quad (19)$$
$$O_{Fluorescence} = 18 \quad (20)$$
$$O_{Human} = 20.5 \quad (21)$$

The calculations yield :

[1] 11.725 GHz according to label on receiver

$$\begin{aligned}
T_{EL} &= 174.58 \pm 10.49K & (22)\\
T_{SUN} &= 248.20 \pm 33.19K & (23)\\
K_{prop} &= 0.0423 \pm 0.0021 & (24)\\
\theta_{rsun} &= 0.252 \pm 0.020^\circ & (25)\\
\theta_{rbeam} &\approx 1.5^\circ & (26)\\
\therefore ratio &= 35.4 \pm 2.81 & (27)
\end{aligned}$$

Adjusting the temperature to reflect the ratio of beamwidth occupied by the source yields

$$T_{SUN} = 8786.28 \pm 1872.37^\circ K \quad (28)$$

Incorporating the further correction factors yields

$$T_{SUN} = 16503.70 \pm 3516.96^\circ K \quad (29)$$

5 Discussion

We initially used equation 8 and the corresponding calibration temperatures to discover T_{EL}. This yielded a negative temperature of -474 K, which makes no sense and tends to argue that our initial temperatures for the human body and ground were dubious.

The problems encountered when using the human as a reference temperature could be due to variations of temperature at the surface of the human, in contrast to the constant core temperature. More importantly, the human failed to completely cover the beam of the receiver, so our observed temperature was inaccurate, even in the context of an uncalibrated scale. Despite this, during the experimental process the temperature of the ground showed the highest degree of variability, and was the most contentious quantity we recorded. Since we had the well established background cosmic background temperature of 2.728 $\pm$ 0.004 and more empirical faith in our recorded temperature of a human then in our temperature of the ground, we utilised equation 9.

After calculating the temperature of the sun using equation 10, there were two divergent camps. People attained a temperature around : $T_{SUN} \approx 250$ K or $T_{SUN} \approx 215$ K, depending on whether they used equation 9 or 8.

6 Conclusion

Through the calibration of a raw data collector, and intimate knowledge of the characteristics of the hardware we were using, we were able to derive a plausible approximation for the temperature of the sun.