Investigating a Pulsar : Vela Pulsar

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Abstract

This paper discusses the correction of the Vela ephemeris, the discovery of its characteristic age and the calculation of the interstellar medium's dispersion measure.

1 Introduction

Pulsars are dynamic bodies in a constant state of flux. Each pulsar is described by a distinct ephemeris, comprising the angular displacement, angular velocity and angular acceleration of that pulsar. The ephemeris can be used to predict the period of the pulsar in question. There are continual deviations in the pulsar characteristics described by the ephemeris, and a prolonged period without measurement and adjustment can lead to an insurmountable difference between the ephemeris and the pulsar it describes. By monitoring and tracking the ephemeris of a pulsar on a daily basis we can observe larger glitches in the periodicity of the pulsar, which are caused by vortices within the pulsar transferring momentum to the crust of the pulsar. These glitches result in sudden speed changes proportional to one 10^6 th of its speed. The characteristic age can be calculated from the components of the ephemeris. The information received from a known pulsar source can be utilised to calculate the dispersion measure of the interstellar medium.

2 Theory

The ephemeris of a pulsar is give by ν, $\dot{\nu}$, $\ddot{\nu}$ and t_0 Standard pulsar astronomy sees pulsar timing measured relative to the centre of our solar system, the Sun. This reference frame is referred to as Barycentric time, and necessitates the adjustment of measurements initially relative to our earth bound reference frame.

The number of pulses measured, N, is given by

$$N = \nu(t-t_0) + \frac{1}{2}\dot{\nu}(t-t_0)^2 + \frac{1}{6}\ddot{\nu}(t-t_0)^3 \quad (1)$$

$$Residuals = N - \lfloor N \rfloor \quad (2)$$

Where the residual term reflects error in the predictions forcast by the existing ephemeris. If the ephemeris was correct the residual term would be zero. These errors can be backtracked to the values of N, and beyond to the ephemeris used to calculated N.

$$N + \Delta N = (\nu + \Delta\nu)(t - t_0) + \frac{1}{2}(\dot{\nu} + \Delta\dot{\nu})(t - t_0)^2 + \frac{1}{6}(\ddot{\nu} + \Delta\ddot{\nu})(t - t_0)^3 \quad (3)$$

$$\Delta N = \Delta\nu(t - t_0) + \frac{1}{2}\Delta\dot{\nu}(t - t_0)^2 + \frac{1}{6}\Delta\ddot{\nu}(t - t_0)^3 \quad (4)$$

A cubic polynomial was fitted to the the residual function.

$$R = A(t - t_0)^3 + B(t - t_0)^2 + C(t - t_0) + D \quad (5)$$

$$\Delta\nu = A \quad (6)$$

$$\Delta\dot{\nu} = 2B \quad (7)$$

$$\Delta\ddot{\nu} = 6C \quad (8)$$

Equating the coefficients of equation 5 & 4 yields

$$\Delta\nu = A \quad (9)$$

$$\Delta\dot{\nu} = 2B \quad (10)$$

$$\Delta\ddot{\nu} = 6C \quad (11)$$

The characteristic age of the pulsar (in seconds) can be calculated from the following equation, given by Lyne and Graham-Smith (1990) :

$$\tau = \frac{p}{2\dot{p}} \quad (12)$$

$$= \frac{\nu}{2\dot{\nu}} \quad (13)$$

By swapping from one discrete frequency band to another frequency band in the midst of receiving information from a pulsar it becomes possible to determine the delay between the the arrival times of the relative frequencies. This is achieved by measuring the signal from a pulsar for the duration of one period of the pulsar, and the ratio of the time spent measuring either of the two frequency bands can be adjusted to extract different information. By predominantly favouring one of the frequency bandwidths, the position of the favoured frequencies arrival time can be discovered, and used to discover the correct order of arrival of the respective frequencies. This delay can be used to calculate the dispersion measure of the interstellar medium.

$$\mu = \sqrt{1 - \frac{f_p}{f}^2} \quad (14)$$

$$v_g = c\mu \tag{15}$$

$$\therefore v_g = c\sqrt{1 - \frac{f_p}{f}^2} \tag{16}$$

$$f_p = 8.5kHz\sqrt{(\frac{n_e}{cm^{-3}})} \tag{17}$$

where f is necessarily larger then f_p in order for the wave packet to propergate. and therefore $0 < \mu < 1$. f_p is the plasma frequency, and is defined as the minimum frequency that a plasma will transmit.

$$\Delta t = 4.15 \times 10^6 \times (\frac{1}{f_1^2} - \frac{1}{f_2^2}) \times DM \tag{18}$$

$$\text{Where : } DM = \int_0^{distance} n_e dl \tag{19}$$

$$= n_e \int_0^{distance} dl \tag{20}$$

$$= n_e distance \tag{21}$$

This assumes a homogeneous distribution of electrons between the earth and the pulsar in question, and the distance refers to the distance between the earth and the pulsar in question. The electron density, n_e, is somewhere between 0.03 and 0.08 electrons/cm^3 (Lorimer and Kramer, 2005).

3 Procedure

A hydrogen maser clock was started at the detection of the first pulse. The existing ephemeris was used to calculate the expected period of the pulsar.

The arrival time of the pulses was measured from the observatories perspective, and adjusted to reflect the pulsar astronomy standard of barycentric units. N is calculated by using equation 1. The residuals are calculated through the use of equation 2.

A 3rd order polynomial is fitted to these residuals with the aid of either Excel solver, or with the third order polynomial deriving tools offered within the graphing functions of Microsoft office.

The coefficients of the fitted curve were then used to correct the existing ephemeris. The residuals of this corrected ephemeris were then calculated.

The correct ephemeris then sourced the values of ν and $\dot{\nu}$ used in equation 13 to calculate the characteristic age of the Vela pulsar.

The number of photons received by from the Vela pulsar was measured for one Vela period. 70 % of the scan was taken at 1650 Mhz and the remaining 30 % was taken at 1720 Mhz. This data displayed two prominent peaks, and two gaussians were fitted to this in the same fashion as discussed in Carr (2005). Fitting these gaussians yielded accurate central points to the peaks, and enabled us to calculate the time seperating the peaks accurately. This seperation time was then used to calculate the dispersion measure of the interstellar medium using equation 18.

This dispersion measure, and the rough electron density, were used in equation 21 to produce a rough estimate of the distance to the Vela pulsar.

4 Results

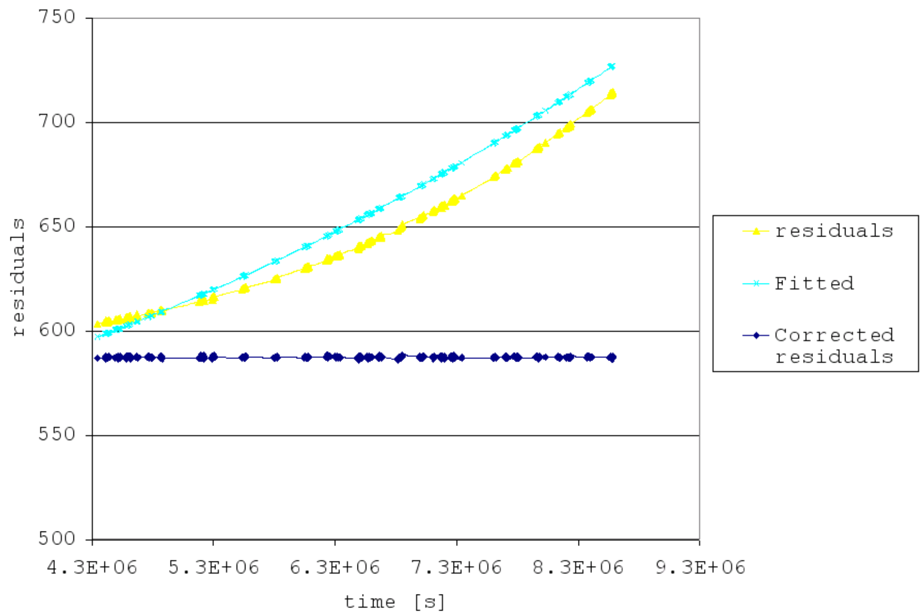

Figure 1: The residuals of the original ephemeris and the corrected ephemeris

Term	original	corrected
ν	11.19267220	11.19267220
$\dot{\nu}$	-1.556227200E-11	-1.556058853E-11
$\ddot{\nu}$	2.518700000E-21	9.209019168E-22

Table 1: Table of original and modified ephemeris

The characteristic age of the Vela pulsar is calculated to be approximately 11400 years.

There is a 0.060570226 second difference in arrival time between the frequency bandwidths. The dispersion measure is calculated to be 498.32 pc cm^{-3} This yields a distance of 16610.65 pc

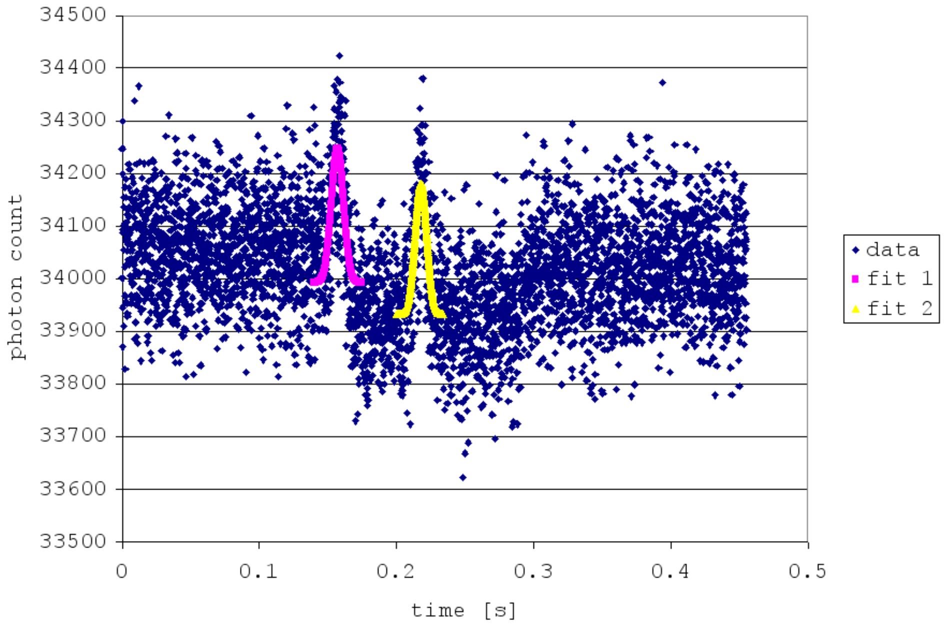

Figure 2: proton count vs time

5 Discussion

Fitting the cubic polynomial to the residuals was non-trivial. Solver seemed incapable of minimising the least squares error in the range we were investigating. We were able to get the error down through the thoughtful manipulation of values, and solver only increased the error with its attempts. Adjusting the sensitivity of solver resolved nothing, and no amount of fiddling could get it to interact pleasantly with the data. There was far more success using the trendline support offered amongst the graphing tools, which offered up the third order polynomial coefficients associated with the original residual data.

The residuals calculated from the corrected ephemeris were non-zero. They were approximately constant however, and correlated closely to the constant discovered when the third order polynomial was fitted to the original residuals.

The dispersion measure for the interstellar medium separating Vela from earth is 480 pc cm^{-3} according to the NE2001 model (Cordes-Lazio, 2002). This is similar to the value produced by our numerical analysis and subsequent utilisation of equation 18.

The calculation of the distance separating earth from Vela is far rougher. Vela is 5000 pc away from earth, and this figure is irreconcilable with our calculated figure. Our assumption of the homogeneity in the electron density of the interstellar medium, as well as the value we adopted as the invariant electron density are misleading. If we maintain our assumption that the electron density is uniform, and use the given distance of 5000 pc and equation 21, then the electron density is calculated to be 0.1 electron cm^{-3}. If we assume that the original electron density was correct, then the deviations could be explain by a region of unusually high electron density between earth and the Vela pulsar.

6 Conclusion

The corrected ephemeris resulted in predictions that correlated strongly to the behaviour observed experimentally. By measuring the arrival times of two different frequency components in a single Vela pulse, we were able to successfully calculate the interstellar dispersion measure. This quantity, coupled with existing theory, volunteered the approximate distance between earth and the Vela pulsar.

References

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Lyne, A. and Graham-Smith, F. (1990), *Pulsar Astronomy*, Cambridge University Press.